

Mathematics
Grade 10
Unified Mathematics IV

Dr. Mark Toback, Superintendent<br>Committee: Cindy Candiano-Schemly<br>Compliance Update Completed June 2022

This curriculum may be modified through varying techniques, strategies, and materials as per an individual student's Individualized Educational Plan (IEP)

# Approved by the Wayne Township Board of Education at the regular meeting held on November 15, 2018. 

New Jersey Student Learning Standards For Mathematics

Standards for Mathematical Practice


#### Abstract

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately) and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).


1 Make sense of problems and persevere in solving them.

Mathematically proficient students:

- explain to themselves the meaning of a problem and looking for entry points to its solution.
- analyze givens, constraints, relationships, and goals.
- make conjectures about the form and meaning of the solution attempt.
- consider analogous problems, and try special cases and simpler forms of the original problem.
- monitor and evaluate their progress and change course if necessary.
- transform algebraic expressions or change the viewing window on their graphing calculator to get information.
- explain correspondences between equations, verbal descriptions, tables, and graphs.
- draw diagrams of important features and relationships, graph data, and search for regularity or trends.
- use concrete objects or pictures to help conceptualize and solve a problem.
- check their answers to problems using a different method.
- ask themselves, "Does this make sense?"
- understand the approaches of others to solving complex problems.

2 Reason abstractly and quantitatively.

Mathematically proficient students:

- make sense of quantities and their relationships in problem situations.
$\checkmark$ decontextualize (abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents and
contextualize (pause as needed during the manipulation process in order to probe into the referents for the symbols involved).
- use quantitative reasoning that entails creating a coherent representation of quantities, not just how to compute them
- know and flexibly use different properties of operations and objects.


## 3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students:

- understand and use stated assumptions, definitions, and previously established results in constructing arguments.
- make conjectures and build a logical progression of statements to explore the truth of their conjectures.
- analyze situations by breaking them into cases
- recognize and use counterexamples.
- justify their conclusions, communicate them to others, and respond to the arguments of others.
- reason inductively about data, making plausible arguments that take into account the context
- compare the effectiveness of plausible arguments
- distinguish correct logic or reasoning from that which is flawed
$\checkmark$ elementary students construct arguments using objects, drawings, diagrams, and actions..
$\checkmark$ later students learn to determine domains to which an argument applies.
- listen or read the arguments of others, decide whether they make sense, and ask useful questions


## 4 Model with mathematics.

Mathematically proficient students:

- apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
$\checkmark$ In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.
$\checkmark$ By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.
- simplify a complicated situation, realizing that these may need revision later.
- identify important quantities in a practical situation
- map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
- analyze those relationships mathematically to draw conclusions.
- interpret their mathematical results in the context of the situation.
- reflect on whether the results make sense, possibly improving the model if it has not served its purpose.


## 5 Use appropriate tools strategically.

Mathematically proficient students:

- consider available tools when solving a mathematical problem.
- are familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools
- detect possible errors by using estimations and other mathematical knowledge.
- know that technology can enable them to visualize the results of varying assumptions, and explore consequences.
- identify relevant mathematical resources and use them to pose or solve problems.
- use technological tools to explore and deepen their understanding of concepts.


## 6 Attend to precision.

Mathematically proficient students:

- try to communicate precisely to others.
- use clear definitions in discussion with others and in their own reasoning.
- state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
- specify units of measure and label axes to clarify the correspondence with quantities in a problem.
- calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the context.
$\checkmark$ In the elementary grades, students give carefully formulated explanations to each other.
$\checkmark$ In high school, students have learned to examine claims and make explicit use of definitions.


## 7 Look for and make use of structure.

Mathematically proficient students:

- look closely to discern a pattern or structure.
$\checkmark$ Young students might notice that three and seven more is the same amount as seven and three more.
$\checkmark$ Later, students will see $7 \times 8$ equals the well-remembered $7 \times 5+7 \times 3$, in preparation for the distributive property.
$\checkmark$ In the expression $x^{2}+9 x+14$, older students can see the 14 as $2 \times 7$ and the 9 as $2+7$.
- step back for an overview and can shift perspective.
- see complicated things, such as some algebraic expressions, as single objects or composed of several objects.


## 8 Look for and express regularity in repeated reasoning.

Mathematically proficient students:

- notice if calculations are repeated
- look both for general methods and for shortcuts.
- maintain oversight of the process, while attending to the details.
- continually evaluate the reasonableness of intermediate results.

| Content Area/ <br> Grade Level/ <br> Course: | Unified Mathematics IV - Grade 10 |
| :--- | :--- |
| Unit Plan Title: | Unit 1 - Sequences and Series |
| Time Frame | 14 days |
| Anchor Standards/Domain* $\quad$ *i.e: ELA: reading, writing i.e.: Math: Algebra |  |
| Funse\| |  |

Functions - Interpreting Functions F-IF
Functions - Building Functions F-BF
Algebra - Seeing Structure in Equations A-SSE
Algebra - Arithmetic with Polynomials and Rational Expressions A-APR

## Unit Overview

- Write sequences from explicit and recursive formulas
- Use summation notation to write the sum of sequences
- Recognize, write, and use arithmetic sequences and geometric sequences
- Find the sum of the first $n$ terms of arithmetic and geometric series
- Find the sum of an infinite geometric series


## Standard Number(s) ${ }^{*}$ i.e: Math: F.LE.A. 4 i.e.: NJSLSA.R4.

- NJSLSF.BF2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
- NJSLSA.SSE4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.
- NJSLSA.APR5. (+) Know and apply the Binomial Theorem for the expansion of $(x+y)^{n}$ in powers of $x$ and $y$ for a positive integer $n$, where $x$ and $y$ are any numbers, with coefficients determined for example by Pascal's Triangle. ${ }^{1}$
- NJSLSF.IF3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0)=f(1)=1, f(n+1)=f(n)+f(n-1)$ for $n \geq 1$.
- 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
- 8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
- 8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original
- and existing algorithms.
- 9.1.12.PB.2: Prioritize financial decisions by considering alternatives and possible consequences.
- 9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
- 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E. 12 prof.CR3.a)
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
- 9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
- RST.9-10.3./RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
- RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
- RST.9-10.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.
- CRP2. Apply appropriate academic and technical skills.
- CRP4. Communicate clearly and effectively and with reason.
- CRP6. Demonstrate creativity and innovation.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP11. Use technology to enhance productivity.
- CRP12. Work productively in teams while using cultural global competence.
- Standards for Mathematical Practice
- K-12.MP.1 Make sense of problems and persevere in solving them.
- K-12.MP. 2 Reason abstractly and quantitatively.
- K-12.MP. 3 Construct viable arguments and critique the reasoning of others.
- K-12.MP. 4 Model with mathematics
- K-12.MP. 5 Use appropriate tools strategically.
- K-12.MP. 6 Attend to precision
- K-12.MP. 7 Look for and make use of structure.
- K-12.MP. 8 Look for and express regularity in repeated reasoning.


## Intended Outcomes - \{Essential Questions\}

- How can we recognize, write, and find the nth terms of sequences?
- In what situations would sequences and series be used to model and solve real life problems?
- How can summation notation be used to represent a series?
- Can you find the sum of an infinite series?


## Enduring Understandings

Students should be able to:

- Recognize patterns in sequences
- Write the rules
- Identify if an infinite series has a sum and know how to find it.

In this unit plan, the following $21{ }^{\text {st }}$ Century themes and skills are addressed.


- Use explicit and recursive rules to find terms of sequences.
- Model and solve real life problems using sequences and series.
- Write a series using summation notation and given summation notation expand the series.
- Calculate sums of infinite series when possible; when not possible justify why an infinite series does not have a sum.

Assessments (Pre, Formative, Summative, Other) Denote required common assessments with an *
Assessments - pretests, formative, summative - are diagnostic tools used to formulate where lessons should begin, where students are as the instruction progresses, what students have mastered, and where they may need to be retaught to attain mastery. Assessments are designed to collect data that will be used to decide the direction in which each class period will proceed. A variety of alternative assessment methods are used for student evaluation. Evaluations may include, but are not limited to, the following:
A. Performance based tasks in support of activities for essential understanding of objectives.
$\begin{array}{ll}\text { I. } & \text { Projects } \\ \text { II. } & \text { Reports }\end{array}$
III. Investigations
IV. Research
B. Other evidence of student learning.
I. Class Participation
II. *Benchmark Tests/Quizzes
III. Teacher Observations
C. Varied types of assessment measures to be employed, including rubrics.
I. Use of communicators
II. Group work
III. Clickers
IV. Exit Cards
V. Homework
VI. Teacher quizzes/tests
VII. $\quad$ Common Core Midterm and Final Examination

Teaching and Learning Activities

| Activities | Teacher Led Instruction <br> Student directed discovery activities <br> Online Videos/Powerpoint Presentations <br> Integrate the TI-84 Calculator <br> Guided and Independent practice <br> Collaborative Learning <br> Desmos |
| :---: | :--- |
| Differentiation Strategies | $\bullet$ Leveled practice problems <br> $\bullet$ - Group work with pairing students with similar levels <br> $\bullet$ Group work with pairing students of varying levels |


|  | Differentiation Strategies for Special Education Students <br> Differentiation Strategies for Gifted and Talented Students |
| :--- | :--- |
| Differentiation Strategies for ELL Students <br> Differentiation Strategies for At Risk Students |  |
| Honors |  |
| Resources |  |

- Algebra 2 and Trigonometry revised edition (Dolciani) Publisher: Houghton Mifflin. 1983.
- Algebra 2 (Larson); Publisher: McDougal Littell. 2001.
- Precalculus with Limits: A Graphing Approach. 3rd ed (Larson)

Publisher: Houghton Mifflin. 2001.

- Advanced Mathematics: Precalculus with Discrete Mathematics and Data Analysis (Brown) Publisher: McDougal Littell, Houghton Mifflin
- Algebra 2 - McDougal Littell - Teacher's Resource
- Precalculus with Limits : A Graphing Approach - Houghton Mifflin - Teacher’s Resource
- Advanced Mathematics : Precalculus with Discrete Mathematics and Data Analysis - Teacher's Resource
- Supplemental Materials through ClassZone
- Extra Practice Masters
- Test/Quiz Masters
- Reteaching Masters
- Enrichment Activities
- Graphing Calculator
- Desmos
- http://explorelearning.com
- http://education.ti.com
- http://illustrativemathematics.org/standards/hs
- www.brightstorm.com
- www.khanacademy.com
- http://www.corestandards.org/assets/CCSSI_Mathematics_Appendix_A.pdf
- http://nlvm.usu.edu/en/nav/grade g 4.html

| Content Area/ Grade Level/ Course: | Unified Mathematics IV - Grade 10 |
| :---: | :---: |
| Unit Plan Title: | Unit 2 - Functions and Their Graphs |
| Time Frame | 20 Days |
| Anchor Standards/Domain* *i.e: ELA: reading, writing i.e.: Math: Algebra |  |
| Functions - Building Functions - F.BF <br> Functions - Interpreting Functions - F.IF |  |
| Unit Overview |  |
| - Evaluate <br> - Analyze <br> - Interpret <br> - Recogniz <br> - Identify <br> - Find arit <br> - Find inv | ctions and find their domains <br> phs of functions <br> find formulas for piecewise defined functions <br> ven and odd functions using equations and graphs <br> graph shifts, reflections, and non-rigid transformations of functions <br> tic combinations and compositions of functions <br> s of functions graphically and algebraically |
| Standard Number(s) * i.e: Math: F.LE.A. 4 i.e.: NJSLSA.R4. |  |
| - NJSLSF.BF3 Identify the effect on the graph of replacing $f(x)$ by $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. <br> - NJSLSF.IF1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. <br> The graph of $f$ is the graph of the equation $y=f(x)$. <br> - NJSLSF.IF2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. <br> - NJSLSF.IF4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* |  |

- NJSLSF.IF7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ${ }^{\star}$
a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- NJSLSF.BF4. Find inverse functions.
a. Solve an equation of the form $f(x)=c$ for a simple function $f$ that has an inverse and write an expression for the inverse.

For example, $f(x)=2 x^{3}$ or $f(x)=(x+1) /(x-1)$ for $\boldsymbol{x} \neq 1$.
b. $(+)$ Verify by composition that one function is the inverse of another.
c. $(+)$ Read values of an inverse function from a graph or a table, given that the function has an inverse.
d. $(+)$ Produce an invertible function from a non-invertible function by restricting the domain.

- 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
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## Intended Outcomes - \{Essential Questions\}

- How can relations and functions be represented?
- What characteristics can be used to describe a function's behavior?
- How do changes to an equation affect the graph?
- What are the different ways we can combine functions and how does it affect the domain?
- How do functions and their inverses relate to each other?
- Does every function have an inverse function?


## Enduring Understandings

Students should be able to:

- Graph functions and identify their characteristics.
- Understand the relationship between a function and its inverse.

In this unit plan, the following $\mathbf{2 1}^{\text {st }}$ Century themes and skills are addressed.


Students will be able to:

- Use multiple representations for relations and functions.
- Identify relations that can be classified as functions.
- Use function notation to evaluate when given an equation or graph.
- Evaluate difference quotients.
- Find the Domain of Functions from various representations
- Graph, write the rule for, and evaluate piecewise functions.
- Determine when a function increasing, decreasing, and constant. Represent the intervals using interval notation.
- Classify functions as even, odd, or neither from graphs, equations, or tables.
- Graph common functions, including quadratic, cubic, square root, cube, root, and absolute value.
- Graph rigid and non-rigid transformations of functions when provided with graph or equation.
- Represent transformations of graphs using function notation
- Perform combination and compositions of functions from graphs, equations, or tables.
- Find Domains of compositions of functions
- Express a function as the composition of two functions
- Find inverse function
- Verify that two functions are inverses using graphs, equations, and tables.
- Classify functions as one-to-one by using the horizontal line test.
- Understand how the one-to-oneness relates to the existence of an inverse function.
- Select an appropriate portion of the domain to force the functions inverse to be a function.

Assessments (Pre, Formative, Summative, Other) Denote required common assessments with an *
Assessments - pretests, formative, summative - are diagnostic tools used to formulate where lessons should begin, where students are as the instruction progresses, what students have mastered, and where they may need to be retaught to attain mastery. Assessments are designed to collect data that will be used to decide the direction in which each class period will proceed. A variety of alternative assessment methods are used for student evaluation. Evaluations may include, but are not limited to, the following:
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II. Reports
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B. Other evidence of student learning.
I. Class Participation
II. *Benchmark Tests/Quizzes
III. Teacher Observations
C. Varied types of assessment measures to be employed, including rubrics.
I. Use of communicators
II. Group work
III. Clickers
IV. Exit Cards
V. Homework
VI. Teacher quizzes/tests
VII. *Common Core Midterm and Final Examination

Teaching and Learning Activities

| Activities | Teacher Led Instruction <br> Student directed discovery activities <br> Online Videos/Powerpoint Presentations <br> Integrate the TI-84 Calculator <br> Guided and Independent practice <br> Collaborative Learning <br> Group activity/presentation on identifying characteristics of a function given the <br> rule. |
| :---: | :--- |
| Differentiation Strategies | • Leveled practice problems <br> $\bullet$ <br> $\bullet$ <br> - Group work with pairing students with similar levels |


|  | Differentiation Strategies for Special Education Students <br> Differentiation Strategies for Gifted and Talented Students <br> Differentiation Strategies for ELL Students |
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|  | Differentiation Strategies for At Risk Students |
| Honors |  |

## Resources

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- Enrichment Activities
- Graphing Calculator
- http://explorelearning.com
- http://education.ti.com
- http://illustrativemathematics.org/standards/hs
- www.brightstorm.com
- www.khanacademy.com
- http://www.corestandards.org/assets/CCSSI Mathematics Appendix A.pdf
- http://nlvm.usu.edu/en/nav/grade_g_4.html

| Content Area/ <br> Grade Level/ <br> Course: | Unified Mathematics IV - Grade 10 |
| :--- | :--- |
| Unit Plan Title: | Unit 3 - Polynomial and Rational Functions |
| Time Frame | 18 or 25* days |
| Anchor Standards/Domain* $\quad$ *i.e: ELA: reading, writing i.e.: Math: Algebra $^{\text {Number and Quantity - The Complex Number System N-CN }}$ |  |
| Algebra - Reasoning with Equations and Inequalities - A.REI |  |
| Functions - Interpreting Functions F-IF |  |
| Algebra - Seeing Structure in Equations A-SSE |  |
| Algebra - Arithmetic with Polynomials and Rational Expressions A-APR |  |
| Unit Overview |  |

- Sketch and analyze graphs of quadratic and polynomial functions
- Use the Rational Root Theorem, Factor Theorem, and the Fundamental Theorem of Algebra to find the real and complex zeros of polynomial functions of higher degree
- *Perform operations on rational expressions.
- *Solve rational equations.
- *Determine the domain, find asymptotes, locations of holes, and sketch the graphs of rational functions


## Standard Number(s) ${ }^{*}$ i.e: Math: F.LE.A. 4 i.e.: NJSLSA.R4.

- NJSLSN.CN1. Know there is a complex number $i$ such that $i^{2}=-1$, and every complex number has the form $a+b i$ with $a$ and $b$ real.
- NJSLSN.CN2. Use the relation $i^{2}=-1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
- NJSLSN.CN3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
- NJSLSN.CN5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1+\sqrt{ } 3 i)^{3}=8$ because $(-1+\sqrt{ } 3 i)$ has modulus 2 and argument $120^{\circ}$.
- NJSLSN.CN6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
- NJSLSN.CN7. Solve quadratic equations with real coefficients that have complex solutions.
- NJSLSN.CN8. (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^{2}+4$ as $(x+2 i)(x-2 i)$.
- NJSLSN.CN9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
- NJSLSA.SSE1. Interpret expressions that represent a quantity in terms of its context.
a. Interpret parts of an expression, such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^{n}$ as the product of $P$ and a factor not depending on $P$.
- NJSLSA.SSE2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^{4}-y^{4}$ as $\left(x^{2}\right)^{2}-\left(y^{2}\right)^{2}$, thus recognizing it as a difference of squares that can be factored as $\left(x^{2}-y^{2}\right)\left(x^{2}+y^{2}\right)$.
- NJSLSA.SSE3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
a. Factor a quadratic expression to reveal the zeros of the function it defines.
b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- NJSLSA.APR2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$, the remainder on division by $x-a$ is $p(a)$, so $p(a)=0$ if and only if $(x-a)$ is a factor of $p(x)$.
- NJSLSA.APR3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- NJSLSA.APR4. Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $\left(x^{2}+y^{2}\right)^{2}=\left(x^{2}-y^{2}\right)^{2}+(2 x y)^{2}$ can be used to generate Pythagorean triples.
- NJSLSA.APR6. Rewrite simple rational expressions in different forms; write ${ }^{a(x)} / b(x)$ in the form $q(x)+$ ${ }^{r(x)}{ }_{b(x)}$, where $a(x), b(x), q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
- NJSLSA.APR7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
- NJSLSA.REI1 Explain each step in solving simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- NJSLSA.REI2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- NJSLSA.REI4. Solve quadratic equations in one variable.
b. Solve quadratic equations by inspection (e.g. for $x^{2}=49$ ), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm b i$ for real number $a$ and $b$.
- NJSLSA.REI7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y=$ $-3 x$ and the circle $x^{2}+y^{2}=3$.
- NJSLSA.REI11. Explain why the $x$-coordinates of the points where the graphs of the equations $y=f(x)$ and $y=g(x)$ intersect are the solutions of the equation $f(x)=g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and /or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- NJSLSF.IF4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- NJSLSF.IF7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are
available, and showing end behavior.
- NJSLSF.IF8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- NJSLSF.IF9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
- NJSLSA-CED1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions
- 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
- 8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
- 8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original
- and existing algorithms.
- 9.1.12.PB.2: Prioritize financial decisions by considering alternatives and possible consequences.
- 9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
- 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E. 12 prof.CR3.a)
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
- 9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
- RST.9-10.3./RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
- RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
- RST.9-10.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.
- CRP2. Apply appropriate academic and technical skills.
- CRP4. Communicate clearly and effectively and with reason.
- CRP6. Demonstrate creativity and innovation.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP11. Use technology to enhance productivity.
- CRP12. Work productively in teams while using cultural global competence.
- Standards for Mathematical Practice
- K-12.MP.1 Make sense of problems and persevere in solving them.
- K-12.MP. 2 Reason abstractly and quantitatively.
- K-12.MP. 3 Construct viable arguments and critique the reasoning of others.
- K-12.MP. 4 Model with mathematics
- K-12.MP. 5 Use appropriate tools strategically.
- K-12.MP. 6 Attend to precision
- K-12.MP. 7 Look for and make use of structure.
- K-12.MP. 8 Look for and express regularity in repeated reasoning.


## Intended Outcomes - \{Essential Questions\}

- How do we solve equations that have no real solution?
- What methods can we use to find zeros of polynomial functions?
- How can we use equations of quadratic, polynomial, and rational functions to determine major characteristics of the functions' graphs?
- What methods can be used to simplify rational expressions and perform operations on rational expressions?
- What methods can be used to solve rational equations?


## Enduring Understandings

Students should be able to:

- Graph quadratic, polynomial, and rational functions.
- Find all zeros of polynomial functions.
- Simplify rational expressions and solve rational equations.

In this unit plan, the following $\mathbf{2 1}^{\text {st }}$ Century themes and skills are addressed.

|  | Check all that apply. <br> $21^{\text {st }}$ Century Themes | Indicate whether these skills are E-Encouraged, T-Taught, or A-Assessed in this unit by marking E, T, A on the line before the appropriate skill. <br> $21^{\text {st }}$ Century Skills |  |
| :---: | :---: | :---: | :---: |
| X | Global Awareness <br> Environmental Literacy <br> Health Literacy <br> Civic Literacy <br> Financial, Economic, Business, and <br> Entrepreneurial Literacy | E | Creativity and Innovation |
|  |  | ETA | Critical Thinking and Problem Solving |
|  |  | ET | Communication |
|  |  | E | Collaboration |
| X |  |  |  |
| Student | rning Targets/Objectives (Stude | now/Stud | nts will understand) |

Students will be able to:

- Determine intercepts and extrema and graph quadratic functions given in standard, vertex, and intercept form.
- Write equations of quadratic functions.
- Use quadratic equations to model real-life situations.
- Determine real zeros and end behavior of polynomial functions and use the characteristics to graph.
- Find all zeros of polynomial functions using appropriate methods including factoring, polynomial division, quadratic formula, analysis of graph, and rational zeros test.
- Determine zeros, intercepts, and asymptotes of rational functions and use the characteristics to graph.
- Simplify complex fractions.
- Add, subtract, multiply, and divide rational expressions.
- Solve rational equations and check for extraneous solutions.


## Assessments (Pre, Formative, Summative, Other) Denote required common assessments with an *

Assessments - pretests, formative, summative - are diagnostic tools used to formulate where lessons should begin, where students are as the instruction progresses, what students have mastered, and where they may need to be retaught to attain mastery. Assessments are designed to collect data that will be used to decide the direction in which each class period will proceed. A variety of alternative assessment methods are used for student evaluation. Evaluations may include, but are not limited to, the following:
A. Performance based tasks in support of activities for essential understanding of objectives.

| I. Projects <br> II. Reports <br> III. Investiga <br> IV. Research | ns |
| :---: | :---: |
| B. Other evidence of student learning. <br> I. Class Participation <br> II. *Benchmark Tests/Quizzes <br> III. Teacher Observations |  |
| C. Varied types of asse <br> I. <br> Use of c <br> II. Group w <br> III. Clickers <br> IV. Exit Car <br> V. Homewo <br> VI. Teacher <br> VII. *Commo | ent measures to be employed, including rubrics. municators <br> zzes/tests <br> Core Midterm and Final Examination |
| Teaching and Learning Activities |  |
| Activities | Teacher Led Instruction <br> Student directed discovery activities <br> Online Videos/Powerpoint Presentations <br> Integrate the TI-84 Calculator <br> Guided and Independent practice <br> Collaborative Learning <br> Desmos <br> Group project where students create polynomial and rational equations that satisfy given conditions. |
| Differentiation Strategies | - Leveled practice problems <br> - Group work with pairing students with similar levels <br> - Group work with pairing students of varying levels <br> Differentiation Strategies for Special Education Students Differentiation Strategies for Gifted and Talented Students Differentiation Strategies for ELL Students Differentiation Strategies for At Risk Students |
| Honors |  |
| Resources |  |
| - Algebra 2 and Trigonometry revised edition (Dolciani); Publisher: Houghton Mifflin. 1983. |  |

- Algebra 2 (Larson) ; Publisher: McDougal Littell. 2001.
- Precalculus with Limits: A Graphing Approach. 3rd ed (Larson)

Publisher: Houghton Mifflin. 2001.

- Advanced Mathematics: Precalculus with Discrete Mathematics and Data Analysis (Brown)

Publisher: McDougal Littell, Houghton Mifflin

- Algebra 2 - McDougal Littell - Teacher's Resource
- Precalculus with Limits : A Graphing Approach - Houghton Mifflin - Teacher’s Resource
- Advanced Mathematics : Precalculus with Discrete Mathematics and Data Analysis - Teacher's Resource
- Supplemental Materials through ClassZone
- Extra Practice Masters
- Test/Quiz Masters
- Reteaching Masters
- Enrichment Activities
- Graphing Calculator
- Desmos
- http://explorelearning.com
- http://education.ti.com
- http://illustrativemathematics.org/standards/hs
- www.brightstorm.com
- www.khanacademy.com
- http://www.corestandards.org/assets/CCSSI Mathematics_Appendix A.pdf
- http://nlvm.usu.edu/en/nav/grade g 4.html

| Content Area/ <br> Grade Level/ <br> Course: | Unified Mathematics IV - Grade 10 |
| :--- | :--- |
| Unit Plan Title: | Unit 4: Exponential and Logarithmic Functions |
| Time Frame | 16 Days |
| Anchor Standards/Domain* *i.e: ELA: reading, writing i.e.: Math: Algebra |  |
| Functions - Building Functions F-BF |  |
| Functions - Linear, Quadratic, and Exponential Models F-LE |  |
| Algebra - Seeing Structure in Equations A-SSE |  |
| Algebra - Reasoning with Equations and Inequalities - A-REI |  |
| Algebra - Creating Equations A-CED |  |
| Number and Quantity - The Real Number System N-RN |  |
| Number and Quantity - Quantities N-Q |  |
| Unit Overview |  |
| • Recognize, evaluate, and graph exponential and logarithmic functions |  |
| • Use the number $e$ as a base of an exponential function and evaluate natural logarithmic expressions |  |
| • Rewrite logarithmic functions with different bases |  |
| • Use properties of logarithms to evaluate, rewrite, and expand, or condense logarithmic expressions |  |
| • Solve exponential and logarithmic equations |  |
| logarithmic models to solve real-life problems |  |

## Standard Number(s) * i.e: Math: F.LE.A. 4 i.e.: NJSLSA.R4.

- NJSLSN.RN1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{\frac{1}{3}}$ to be the cube root of 5 because we want
$\left(5^{\frac{1}{3}}\right)^{3}=5\left(^{\frac{1}{3}}\right)^{3}$ to hold, so $5\left(^{\frac{1}{3}}\right)^{3}$ must equal 5 .
- NJSLSN.RN2 Rewrite expressions involving radicals rational exponents using the properties of exponents.
- NJSLSN.QA2 Define appropriate quantities for the purpose of descriptive modeling
- NJSLSF.BF1 Write a function that describes a relationship between two quantities.
b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
- NJSLSF.BF3 Identify the effect on the graph of replacing $f(x)$ by $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
- NJSLSF.BF5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
- NJSLSF.LE1 Distinguish between situations that can be modeled with linear functions and with exponential functions.
a. Prove that linear functions grow by equal differences over equal intervals; and that exponential functions grow by equal factors over equal intervals.
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- NJSLSF.LE2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- NJSLSF.LE3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function
- NJSLSF.LE4 Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $a b^{\mathrm{ct}}=d$ where $a, c$, and $d$ are numbers and the base $b$ is 2 , 10 , or $e$; evaluate the logarithm using technology.
- NJSLSF.LE5 Interpret the parameters in a linear or exponential function in terms of a context
- NJSLSA.SSE3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
c. Use the properties of exponents to transform expressions for exponential functions. For example the expression $1.15^{t}$ can be rewritten as $\left(1.15^{1 / 12}\right)^{12 t} \approx 1.012^{12 t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is $15 \%$.
- NJSLSA.REI11. Explain why the $x$-coordinates of the points where the graphs of the equations $y=f(x)$ and $y=g(x)$ intersect are the solutions of the equation $f(x)=g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions
- NJSLSA-CED1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions
- 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
- 8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
- 8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original
- and existing algorithms.
- 9.1.12.PB.2: Prioritize financial decisions by considering alternatives and possible consequences.
- 9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
- 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E. 12 prof.CR3.a)
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
- 9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
- RST.9-10.3./RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
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- RST.9-10.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.
- CRP2. Apply appropriate academic and technical skills.
- CRP4. Communicate clearly and effectively and with reason.
- CRP6. Demonstrate creativity and innovation.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP11. Use technology to enhance productivity.
- CRP12. Work productively in teams while using cultural global competence.
- Standards for Mathematical Practice
- K-12.MP.1 Make sense of problems and persevere in solving them.
- K-12.MP. 2 Reason abstractly and quantitatively.
- K-12.MP. 3 Construct viable arguments and critique the reasoning of others.
- K-12.MP. 4 Model with mathematics
- K-12.MP. 5 Use appropriate tools strategically.
- K-12.MP. 6 Attend to precision
- K-12.MP. 7 Look for and make use of structure.
- K-12.MP. 8 Look for and express regularity in repeated reasoning.


## Intended Outcomes - \{Essential Questions\}

- What is the relationship between exponential and logarithmic functions?
- What techniques can we use to simplify expressions involving logarithms?
- What methods can we use to solve equations involving exponential and logarithmic expressions?
- Why might extraneous solutions occur, and how can we check for them?
- In what types of real-life situations might we use exponential and logarithmic functions to model


## Enduring Understandings

Students should be able to:

- Identify major characteristics of exponential and logarithmic functions.
- Solving equations and real life problem involving exponential and logarithmic expressions.

| Check all that apply. <br> $21^{\text {st }}$ Century Themes |  | Indicate whether these skills are E-Encouraged, $\boldsymbol{T}$-Taught, or A-Assessed in this unit by marking $\mathbf{E}, \mathbf{T}, \mathbf{A}$ on the line before the appropriate skill. <br> 21 ${ }^{\text {st }}$ Century Skills |  |
| :---: | :---: | :---: | :---: |
| X | Global Awareness <br> Environmental Literacy <br> Health Literacy <br> Civic Literacy <br> Financial, Economic, Business, and Entrepreneurial Literacy | E | Creativity and Innovation |
|  |  | ETA | Critical Thinking and Problem Solving |
|  |  | E | Communication |
|  |  | E | Collaboration |
| X |  |  |  |

Students will be able to:

- Determine major characteristics of exponential functions, including domain, range, intercept, and asymptote.
- Graph Exponential Functions and transformations of exponential functions, including the natural base e.
- Determine major characteristics of logarithmic functions, including domain, range, and asymptote
- Use properties of inverses to graph logarithmic functions.
- Graph transformations of logarithmic functions.
- Evaluate Logarithmic expressions by hand and with a graphing utility.
- Use properties of logarithms to condense, expand, and simplify logarithmic expressions.
- Solve equations involving exponential and logarithmic expressions algebraically and graphically.
- Use various techniques including rewriting exponentials to have the same base, changing between exponential and logarithmic form, factoring, and using properties of logs.
- Understand the existence of extraneous solutions and how to check for them.
- Use technology to model and solve real life application problems. Use models including exponential, logarithmic, and logistic. (Growth/Decay)

Assessments (Pre, Formative, Summative, Other) Denote required common assessments with an *
Assessments - pretests, formative, summative - are diagnostic tools used to formulate where lessons should begin, where students are as the instruction progresses, what students have mastered, and where they may need to be retaught to attain mastery. Assessments are designed to collect data that will be used to decide the direction in which each class period will proceed. A variety of alternative assessment methods are used for student evaluation. Evaluations may include, but are not limited to, the following:
A. Performance based tasks in support of activities for essential understanding of objectives.
I. Projects
II. Reports
III. Investigations

| IV. Research |  |
| :---: | :---: |
| B. Other evidence of student learning. <br> I. Class Participation <br> II. *Benchmark Tests/Quizzes <br> III. Teacher Observations |  |
| C. Varied types of assess <br> I. Use of co <br> II. Group wo <br> III. Clickers <br> IV. Exit Card <br> V. Homewor <br> VI. Teacher quize <br> VII. $\quad$ Common | ent measures to be employed, including rubrics. municators <br> zzes/tests <br> Core Midterm and Final Examination |
| Teaching and Learning Activities |  |
| Activities | Teacher Led Instruction <br> Student directed discovery activities <br> Online Videos/Powerpoint Presentations <br> Integrate the TI-84 Calculator <br> Desmos <br> Guided and Independent practice <br> Collaborative Learning <br> Given a data that fits an exponential, logarithmic, or logistic model, use graphing calculator to create a stat plots, find equation of best fit, and use model to make predictions. |
| Differentiation Strategies | - Leveled practice problems <br> - Group work with pairing students with similar levels <br> - Group work with pairing students of varying levels <br> Differentiation Strategies for Special Education Students <br> Differentiation Strategies for Gifted and Talented Students <br> Differentiation Strategies for ELL Students <br> Differentiation Strategies for At Risk Students |
| Honors |  |
| Resources |  |

- Algebra 2 and Trigonometry revised edition (Dolciani) ; Publisher: Houghton Mifflin. 1983.
- Algebra 2 (Larson) ; Publisher: McDougal Littell. 2001.
- Precalculus with Limits: A Graphing Approach. 3rd ed (Larson) Publisher: Houghton Mifflin. 2001.
- Advanced Mathematics: Precalculus with Discrete Mathematics and Data Analysis (Brown) Publisher: McDougal Littell, Houghton Mifflin
- Algebra 2 - McDougal Littell - Teacher's Resource
- Precalculus with Limits : A Graphing Approach - Houghton Mifflin - Teacher’s Resource
- Advanced Mathematics : Precalculus with Discrete Mathematics and Data Analysis - Teacher's Resource
- Supplemental Materials through ClassZone
- Extra Practice Masters
- Test/Quiz Masters
- Reteaching Masters
- Enrichment Activities
- Graphing Calculator
- Desmos
- http://explorelearning.com
- http://education.ti.com
- http://illustrativemathematics.org/standards/hs
- www.brightstorm.com
- www.khanacademy.com
- http://www.corestandards.org/assets/CCSSI Mathematics Appendix A.pdf
- http://nlvm.usu.edu/en/nav/grade g 4.html

| Content Area/ <br> Grade Level/ <br> Course: | Unified Mathematics IV - Grade 10 |
| :--- | :--- |
| Unit Plan Title: | Unit 5: Trigonometry and Circular Functions |
| Time Frame | Trigonometric Functions 40 days <br> Analytic Trigonometry 15 days <br> Additional Topics in Trigonometry 12 days <br> Additional Topics Analytic Geometry 17 days |
| Anchor Standards/Domain* *i.e: ELA: reading, writing i.e.: Math: Algebra |  |

- Functions - Trigonometric Functions - F-TF
- Numbers - The Complex Number System - N-CN


## Unit Overview

Trigonometric Functions

- Convert between radians and degrees, find coterminal angles, and find arc length
- Use trigonometry to find unknown sides or angles of a right triangle
- Identify a unit circle and its relationship to real numbers
- Evaluate trigonometric functions of any angle
- Use fundamental trigonometric identities
- Sketch and analyze graphs of trigonometric functions and their transformations
- Evaluate inverse trigonometric functions
- Use trigonometric functions to model and solve real-life problems


## Analytic Trigonometry

- Use fundamental trigonometric identities to evaluate trigonometric functions and simplify trigonometric expressions
- Verify trigonometric identities algebraically, graphically, and numerically
- Use standard algebraic techniques and inverse trigonometric functions to solve trigonometric equations
- Use sum and difference formulas, multiple-angle formulas, and half-angle formulas to rewrite and evaluate trigonometric functions
Additional Topics in Trigonometry
- Use the Law of Sines and the Law of Cosines to solve oblique triangles
- Find areas of oblique triangles
- Express complex numbers in trigonometric form
- Multiply and divide complex numbers written in trigonometric form
- Apply DeMoivre's Theorem to find powers of complex numbers
- Find n th roots of complex numbers

Additional Topics in Analytic Geometry

- Plot points and find multiple representations of points in the polar coordinate system
- Convert points from rectangular to polar form and vice versa
- Convert equations from rectangular to polar form and vice versa
- Use symmetry to graph polar equations and recognize special polar graphs


## Standard Number(s) * i.e: Math: F.LE.A. 4 i.e.: NJSLSA.R4.

- NJSLSF.TF1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- NJSLSF.TF2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- NJSLSF.TF3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi / 3, \pi / 4$ and $\pi / 6$, and use the unit circle to express the values of sine, cosines, and tangent for $\mathrm{x}, \pi+\mathrm{x}$, and $2 \pi-\mathrm{x}$ in terms of their values for x , where x is any real number.
- NJSLSF.TF4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
- NJSLSF.TF5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
- NJSLSF.TF6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
- NJSLSF.TF7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
- NJSLSF.TF8. Prove the Pythagorean identity $\sin 2(\theta)+\cos 2(\theta)=1$ and use it to find $\sin (\theta), \cos (\theta)$, or $\tan (\theta)$ given $\sin (\theta), \cos (\theta)$, or $\tan (\theta)$ and the quadrant of the angle.
- NJSLSF.TF9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
- NJSLSN.CN4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
- NJSLSN.CN5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example,
$\left(1+\sqrt{ } 3\right.$ i) $3=8$ because $(1+\sqrt{ } 3 i)$ has modulus 2 and argument $120^{\circ}$
- 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
- 8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
- 8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original and existing algorithms.
- 9.1.12.PB.2: Prioritize financial decisions by considering alternatives and possible consequences.
- 9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
- 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E. 12 prof.CR3.a)
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
- 9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
- RST.9-10.3./RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
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- CRP2. Apply appropriate academic and technical skills.
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- Standards for Mathematical Practice
- K-12.MP. 1 Make sense of problems and persevere in solving them.
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- K-12.MP. 3 Construct viable arguments and critique the reasoning of others.
- K-12.MP. 4 Model with mathematics
- K-12.MP. 5 Use appropriate tools strategically.
- K-12.MP. 6 Attend to precision
- K-12.MP. 7 Look for and make use of structure.
- K-12.MP. 8 Look for and express regularity in repeated reasoning.


## Intended Outcomes - \{Essential Questions\}

Trigonometric Functions

- What are the units that can be used to measure angles?
- What is one radian?
- How are the values of the trigonometric functions on the unit circle derived?
- How can the trigonometric functions of acute angles be evaluated?
- How do reference angles allow us to evaluate angles of any measure?
- How do sketches and transformations of sine and cosine graphs relate to their parent functions and graphs?
- How are the graphs of sine and cosine used to derive the graphs of the other four trigonometric functions?
- What restrictions can be placed on the domains of sine, cosine, and tangent so that they are one to one and their inverses exist?
- How are the inverse trigonometric functions and compositions of trigonometric functions evaluated?
- How can trigonometric functions be applied to real life problems?

Analytic Trigonometry

- How are the fundamental trigonometric identities used to evaluate trigonometric functions, simplify trigonometric expressions and rewrite trigonometric expressions?
- How are trigonometric identities verified?
- How do you use standard algebraic techniques and inverse trig functions to solve trigonometric equations?
- How do sum and difference formulas enable one to evaluate trig functions, verify identities and to solve trigonometric equations?
- How are sum and difference formulas, multiple angle formulas, power reducing formulas and product and sum formulas used to simplify, solve, or evaluate trigonometric functions?


## Additional Topics in Trigonometry

- Under what conditions are the Law of Sines and Law of Cosines used?
- How is the "ambiguous" case solved?
- What are all the ways to find the area of a triangle?
- How is the "absolute value" of a complex number found and what are the "modulus and argument"?
- How and why are complex numbers in trigonometric form multiplied and divided?
- How is DeMoivre's Theorem used to find powers and nth roots of complex numbers?
- How can a graphing utility be used to find the nth roots of a complex number?


## Additional Topics in Analytic Geometry

- How are points plotted in the polar coordinate system?
- Explain multiple representations of points in the polar coordinate system?
- How and why are points and equations converted from rectangular to polar form and vice versa?
- How do we graph a polar equation by hand?
- How are symmetry, zeros, and maximum " $r$ " values helpful as graphing aids?
- What are the names of the special polar graphs and how can we identify them?


## Enduring Understandings

Students should be able to:

- Reason mathematically
- Think critically and solve real world problems
- Understanding the unit circle and the relationship to real numbers
- Graphing Trigonometric functions and identifying their characteristics
- Simplifying and Solving Trigonometric expressions and equations
- Understand the relationship between polar and rectangular equations and graphs
- Write, graph, and analyze conics

In this unit plan, the following $21^{\text {st }}$ Century themes and skills are addressed.


Students will be able to:

- Define one radian
- Describe an angle and convert between degree and radian measure
- Identify a unit circle and its relationship to real numbers
- Evaluate trigonometric functions of any angle
- Apply fundamental trigonometric identities
- Sketch the graphs of trigonometric functions
- Evaluate inverse trigonometric functions
- Evaluate the composition of trigonometric functions
- Apply trigonometric functions to model and solve real-life problems
- Apply the fundamental trigonometric identities to evaluate trigonometric functions and simplify trigonometric expressions
- Verify trigonometric Identities
- Prove and apply sum and difference formulas and multiple-angle formulas
- Solve trigonometric equations
- Prove and apply the Law of Sines and the Law of Cosines to solve oblique triangles
- Prove and find the area of oblique triangles
- Multiply and divide complex numbers written in trigonometric form
- Find powers and nth roots of complex numbers
- Plot points and find multiple representations of points in the polar coordinate system
- Convert points from rectangular to polar form and vice versa
- Convert equations from rectangular to polar form and vice versa
- Graph polar equations and recognize special polar graphs


## Assessments (Pre, Formative, Summative, Other) Denote required common assessments with an *

Assessments - pretests, formative, summative - are diagnostic tools used to formulate where lessons should begin, where students are as the instruction progresses, what students have mastered, and where they may need to be retaught to attain mastery. Assessments are designed to collect data that will be used to decide the direction in which each class period will proceed. A variety of alternative assessment methods are used for student evaluation. Evaluations may include, but are not limited to, the following:
A. Performance based tasks in support of activities for essential understanding of objectives.
I. Projects
II. Reports
III. Investigations
IV. Research
B. Other evidence of student learning.
I. Class Participation
II. *Benchmark Tests/Quizzes
III. Teacher Observations
C. Varied types of assessment measures to be employed, including rubrics.
I. Use of communicators
II. Group work
III. Clickers
IV. Exit Cards
V. Homework
VI. Teacher quizzes/tests
VII. $\quad$ Common Core Midterm and Final Examination

## Teaching and Learning Activities

| Activities | Teacher Led Instruction |
| :--- | :--- |
|  | Student directed discovery activities |
|  | Online Videos/Powerpoint Presentations |
|  | Integrate the TI-84 Calculator |
| Desmos |  |
|  | Guided and Independent practice |
|  | Collaborative Learning |
|  | Group activity/presentation on real-life applications of trigonometric functions. |


| Differentiation Strategies | - Leveled practice problems <br> - Group work with pairing students with similar levels <br> - Group work with pairing students of varying levels <br> Differentiation Strategies for Special Education Students <br> Differentiation Strategies for Gifted and Talented Students <br> Differentiation Strategies for ELL Students <br> Differentiation Strategies for At Risk Students |
| :---: | :---: |
| Honors |  |
| Resources |  |

- Algebra 2 and Trigonometry revised edition (Dolciani); Publisher: Houghton Mifflin. 1983.
- Algebra 2 (Larson) ; Publisher: McDougal Littell. 2001.
- Precalculus with Limits: A Graphing Approach. 3rd ed (Larson) ; Publisher: Houghton Mifflin. 2001.
- Advanced Mathematics: Precalculus with Discrete Mathematics and Data Analysis (Brown) Publisher: McDougal Littell, Houghton Mifflin
- Algebra 2 - McDougal Littell - Teacher's Resource
- Precalculus with Limits : A Graphing Approach - Houghton Mifflin - Teacher's Resource
- Advanced Mathematics : Precalculus with Discrete Mathematics and Data Analysis - Teacher's Resource
- Supplemental Materials through ClassZone
- Extra Practice Masters
- Test/Quiz Masters
- Reteaching Masters
- Enrichment Activities
- Graphing Calculator
- Desmos
- http://explorelearning.com
- http://education.ti.com
- http://illustrativemathematics.org/standards/hs
- www.brightstorm.com
- www.khanacademy.com
- http://www.corestandards.org/assets/CCSSI Mathematics_Appendix_A.pdf
- http://nlvm.usu.edu/en/nav/grade g 4.html


| Content Area/ <br> Grade Level/ <br> Course: | Unified Mathematics 4 - Grade 10 |
| :--- | :--- |
| Unit Plan Title: | Unit 6 - Statistics and Probability |
| Time Frame | 11 Days |
| Anchor Standards/Domain* *i.e: ELA: reading, writing i.e.: Math: Algebra |  |
| Statistics and Probability - Interpreting Categorical and Quantitative Data - S-ID <br> Statistics and Probability - Making Inferences and Justifying Conclusions - S-IC <br> Statistics and Probability - Conditional Probability and the Rules of Probability - S-CP |  |
| Unit Overview <br> Students last formally studied probability in Grade 7, when they found probabilities of simple and compound <br> events and designed and used simulations. This unit builds on these concepts, as well as fundamental counting <br> principles and the notion of independence, to develop rules for probability and conditional probability. |  |

## Standard Number(s) $\quad$ *i.e: Math: F.LE.A. $4 \quad$ i.e.: NJSLSA.R4.

- NJSLSS-ID4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
- NJSLSS-ID6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
- NJSLSS-IC1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
- NJSLSS-IC2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5 . Would a result of 5 tails in a row cause you to question the model?
- NJSLSS-IC3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
- NJSLSS-CP1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
- 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
- 8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
- 8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original
- and existing algorithms.
- 9.1.12.PB.2: Prioritize financial decisions by considering alternatives and possible consequences.
- 9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
- 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E. 12 prof.CR3.a)
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
- 9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
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## Intended Outcomes - \{Essential Questions\}

- What is the difference between experimental and theoretical probability?
- What is the difference between combinations and permutations?
- Can mutually exclusive events be independent?
- How can probability models help analyze situations?
- What is the difference between dependent and independent events?
- How does conditional probability affect the overall outcome?


## Enduring Understandings

- You can use multiplication to quickly count the number of ways certain things happen.
- The probability of an impossible event is 0 . The probability of a certain event is 1 . Otherwise the probability of an event is between 0 and 1 .
- To find the probability of two events occurring together, you have to decide whether one event affects the other event.
- Conditional probability exists when two events are dependent.
- You can use probability models to analyze situations and make fair decisions.

In this unit plan, the following $\mathbf{2 1}^{\text {st }}$ Century themes and skills are addressed.

| Check all that apply. <br> 21 ${ }^{\text {st }}$ Century Themes |  | Indicate whether these skills are E-Encouraged, $\boldsymbol{T}$-Taught, or A-Assessed in this unit by marking E, T, A on the line before the appropriate skill. <br> 21 ${ }^{\text {st }}$ Century Skills |  |
| :---: | :---: | :---: | :---: |
| X | Global Awareness <br> Environmental Literacy <br> Health Literacy | E | Creativity and Innovation <br> Critical Thinking and Problem Solving <br> Communication |
|  |  | ETA |  |
|  |  | ET |  |


|  | Civic Literacy |  |
| :--- | :--- | :--- | :--- |
| X | Financial, Economic, Business, and <br> Entrepreneurial Literacy | Collaboration |

## Student Learning Targets/Objectives (Students will know/Students will understand)

Students will be able to:

- Identify the subset of outcomes from the sample space corresponding to the union or intersection of two events.
- Calculate the probability of events defined in terms of unions, intersections, and complements for a simple chance experiment with equally likely outcomes.
- Calculate probabilities given a two-way table of data.
- Use two-way tables to determine if two events are independent.
- Interpret probabilities, including conditional probabilities, in context.
- Given a chance experiment with equally likely outcomes, calculate counts and probabilities by adding or subtracting given counts or probabilities.
- Use the complement rule to calculate the probability of the complement of an event and the multiplication rule for independent events to calculate the probability of the intersection of two independent events.
- Use the addition rule to calculate the probability of a union of two events.
- Use the mean and standard deviation to describe center and variability for a data distribution that is approximately symmetric.
- Recognize when it is reasonable and when it is not reasonable to use a normal curve as a model for a given data distribution.
- Use a calculator to estimate areas under the normal curve.
- Given data, create scatter plots on a graphing calculator, choose an appropriate model, and determine the equation of best fit.
- Distinguish between observational studies, surveys, and experiments.
- Explain why random selection is an important consideration in observational studies and surveys.
- Recognize statistical questions that are answered by estimating a population mean or a population proportion.
- Compare two experiments and determine if the differences between parameters are significant.
- Given conclusions made from an experiment, evaluate validity of the conclusions.

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IV. Exit Cards
V. Homework
VI. Teacher quizzes/tests
VII. $\quad$ Common Core Midterm and Final Examination

Teaching and Learning Activities

| Activities | Math Academy: Are You Game? Explorations in Probability www.actuarialfoundation.org/pdf/Math-Academy-Are-You-Game.pdf <br> Probability Activities <br> www.math.wichita.edu/history/activities/prob-act.html <br> Probability Activities \& Games for Kids <br> http://www.education.com/activity/probability-statistics/ <br> What Are My Chances? - Illuminations <br> http://illuminations.nctm.org/Lesson.aspx?id=2895 <br> Interactivate: Experimental Probability - Shodor <br> http://www.shodor.org/interactivate/activities/ExpProbability/ <br> Conditional Probability and Probability of Simultaneous Events <br> http://www.shodor.org/interactivate/lessons/ConditionalProb/ <br> Conditional Probability Activity <br> http://www.regentprep.org/regents/math/Algebra/APR3/Tconditional.htm |
| :---: | :---: |
| Differentiation Strategies | - Leveled practice problems <br> - Group work with pairing students with similar levels |



